Match-pointing boards with an unequal number of scores
The Neuberg formula

1. Introduction

Consider the following situation. The last table to finish play for the evening are just about to start their final board. Everyone else has finished, and the scoring has been completed save for the final board.

Pair 'X' currently have a top on the final board – but there is still one result to come.

What do you think the chances are that Pair 'X' will still have a top once the last table have finished?

Well, there are three possibilities. Let us say that it is an 11-table complete movement, so a top on a board is 20 match-points.

The three possibilities are:-

- the final table will beat pair 'X's result, so pair 'X' will score only 18 points out of 20.
- The final table will get the same result, so pair 'X' will score 19 points.
- The final table will get a worse result, so pair 'X' will indeed score their complete top and get 20 points.

So, if all three outcomes are equally likely, pair 'X' would have a normal expectancy of 19 points out of 20 in such a situation (the average of 18, 19 and 20).

However, demonstrably not all three outcomes are equally likely. After all, pair 'X' have already beaten 9 out of 9 other results, so it must be heavily odds-on that they will beat the 10th and final result as well – not certain, but very likely.

So, their normal expectancy in such a situation must be closer to 20 points than it is to 19. We will return to this question later.

2. Boards with unequal tops

Of course, in the actual example quoted above, pair 'X' must simply wait until the last table have finished before they can know their real score on the board.

But what happens if the tournament director tells the last table that they cannot play the board because they are too slow? 'Take an average', he says to the last table.

Now we have a real problem. What score to give pair 'X', or indeed all the other pairs who have already played this board?

3. Three possible approaches to the problem

We have seen three different solutions proposed over the years – and they all generate a slightly different final result and therefore potentially a different overall winner as well!

(a) Insert an average into the results, so the top becomes 19 and the bottom becomes 1 (and the average is still 10). The ten actual scores are match-pointed in the normal way 19, 17, 15, 13, 11, 9, 7, 5, 3, 1.

This has the merit of recognising a very important principle in pairs play, which is that all boards should be equally significant (I'll give an extreme example later of why this is important). It also has the merit of simplicity and is the method which has been used by experienced club scorers for years in the pre-computer age.

However, it has the considerable de-merit of being blatantly unfair! Our poor pair 'X' now get only 19 points on the board, which we have already demonstrated is not enough.

(b) Because there are only 10 results, score this board with a top of 18 and express everyone's final result as a percentage of the maximum score which was available to them.

Superficially, this sounds fine, as pair 'X' are getting 100% on the board, which is certainly closer to the mark than the 95% they got under method (i).

However, consider the following rather extreme case. Take a tournament where there are 26-boards in play, of which everyone plays exactly 25 (this just happens to be a convenient number to use). Say it was a very large event – several sections with 51 tables overall and a top of 100.

Pair 'Y' score 50% on all their boards bar one. On one board they have achieved a 65% score. So, (24 x 50) + (1 x 65) = 1265 out of 2500 = 50.6%

However there is something strange about the movement and board 26 is only played in one of the sections and only has 11 results on it. Pair 'Y' have never played this board, so their final score is not affected. Of course, our poor pair 'X' have played the board.

Pair 'X' have also scored 50% on all their boards bar one. On one board they have scored a complete top – and wouldn't you know it, it's board 26: the only one with 11 results!

So method (ii) would give them (24 x 50) + (1 x 20) = 1220 out of 2420 = 50.41%.

Oops!

Pair 'Y' have beaten pair 'X'. Is this fair? Is this what you thought or expected would happen?

Their results are identical save that pair 'Y' have a 65% score on one board and pair 'X' have a 100% on one board. Yet pair 'Y' are the winners?

So, what has gone wrong? Well, what's happened is that the significance of board 26 has become almost irrelevant because it has been played less often that all the other boards.

This is why the basic philosophy contained in approach (ii) is that all boards should be equally significant is very important.

(c) So, there are serious flaws in both the above approaches. Moreover, we need to find a solution to the problem as it really is most unsatisfactory that two (or even three) different and perfectly competent scores might produce two (or even three) different winners given the same set of data.

To the rescue comes Gerard Neuberg of France.

4. The Neuberg principle

I'm afraid that this is where life begins to get complicated in which case I would urge the reader to just trust us and leap to section 6 or beyond!

This said, let us return to our original example – the board with 10 results where 11 results were expected. The correct (i.e. mathematically sound) approach to the problem is as follows:-

(a) We were expecting 11 results, but in fact we have only 10. The ratio of 11/10 is 1.1.

(b) So, assume that each of the 10 results have occurred 1.1 times (rather than only once) and match-point accordingly.

(c) Example:

N/S Score	***Actual Frequency***	***Factored Frequency***	***Match Points***
+ 520	1	1.1	19.9
+ 500	1	1.1	17.7
+ 490	1	1.1	15.5
+ 480	1	1.1	13.3
+ 460	1	1.1	11.1
+ 450	1	1.1	8.9
+ 430	1	1.1	6.7
+ 420	1	1.1	4.5
+ 400	1	1.1	2.3
- 50	1	1.1	0.1
	Total 10	Total 11	

(d) The match points quoted are out of a theoretical top of 20. The top score of 19.9 is arrived at by subtracting the frequency (1.1 in this case) from 21 (i.e. 1 more than the theoretical top). This is exactly the same process that you use already, possibly without realising it, when match pointing normally. If the best score had occurred only once, you would give it 20 out of 20 (i.e. subtract 1 from 21) – had it occurred twice (a shared top) you would give it 19, and so on. The match points for subsequent scores go down in units of 2.2 rather than 2.0. 2.2 is the sum of the previous frequency (1.1 in this case) and the frequency of the result you are trying to calculate (also 1.1 in this case).

(e) A second example may help to illustrate the principle, this time using whole numbers only. Say a board should have been played 16 times (top = 30), but has actually only been played 8 times. So, the ratio of 16/8 is exactly 2 this time. So, imagine that each of the eight actual results had occurred exactly twice, and match point accordingly.

Let us say that our 8 results are as follows:-

N/S score	***Number of occurrences***	***Normal match points (top of 14)***	***Factored frequency***	***Factored match points (top of 30)***	***Calculation***
+ 490	1	14	2	29	31 -2
+ 460	2	11	4	23	29 - 2 - 4
+ 430	3	6	6	13	23 - 4 - 6
+ 400	1	2	2	5	13 - 6 - 2
- 50	1	0	2	1	5 - 2 - 2
Total	8		16		

You can verify this by match pointing a board yourself in the usual way with two scores of + 490, four of + 460, six of +430, two of +400 and two of - 50. You will end up with match point scores of 29, 23, 13, 5 and 1 for the five different results.

5. *The Neuberg formula*

The actual formula is as follows: -

$$\text{Match Points} = \frac{(M \times E) + (E - A)}{A}$$

Where

- M is the match points considering the scores in isolation (14, 11, 6, 2, 0) in the previous example.
- E is the expected number of scores (16)
- A is the actual number of scores (8)

So, the + 490 above scores $\frac{(14 \times 16) + (16 - 8)}{8} = 29$

6. *Application of the formula*

Boards with a different number of results can arise in a variety of different ways, such as the nature of the movement itself or a table being unable to play a board for whatever reason.

In all such instances, and in an ideal world, the Neuberg formula should be used to calculate the final result.

However, it is acknowledged that there are difficulties, not the least being:-

(a) it is difficult (very difficult) to understand; and

(b) it is even more difficult (though not impossible) to perform such calculations without the aid of a computer.

For this reason, and for this reason alone, the formula has not been written into the Laws of Bridge as a 'must do it this way'. Indeed, for clubs who score manually we would strongly recommend that you do not even attempt to score this way.

The important thing is to have a rule about how you will score such boards, and then keep to it. Whatever you do though, please don't have three different scorers all scoring it a different way each night!

All World Bridge Federation, European Bridge League and English Bridge Union events are scored using the Neuberg formula, and have been for the last couple of decades. All – or nearly all – modern computer scoring software has the Neuberg formula already built-in, so if your club uses a computer it is likely that you too are already using the Neuberg formula without even realising it.

7. *Final example*

We have seen how the formula generates a score of 19.9 out of 20 for our pair 'X' in the question at the very start of this paper. And for the case of the 50.6% (1265 match points) pair who somehow managed to beat the pair with 24 averages and 1 top? Well pair 'X' would score [(20 x 51) + (51 – 11)]/11 = 96.4 on board 26 (out of 100). So their final score would be (24 x 50) + 96.4 = 1296.4 out of 2500 = 51.86%. A clear victory for them, which is precisely how it should be!

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